SKETCH AND INVESTIGATE

Q1 The measure of a central angle in a circle equals the measure of the minor arc it intercepts.

Q2 The measures of the central angle and the major arc it intercepts sum to 360°. (The measure of the major arc is equal to 360° minus the measure of the central angle.)

Q3 The measure of an inscribed angle is always half the measure of the arc it intercepts. (Notice that if you drag C beyond D, the arc measurement no longer shows the measure of the arc intercepted by $\angle CDB$.)

Q4 All inscribed angles intercepting the same arc are congruent.

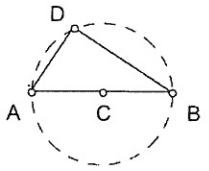
Q5 Every angle inscribed in a semicircle is a right angle.

EXPLORE MORE

15. The expression below is equivalent to the length of the arc. The first part of the expression calculates the fractional part of the circle used by the arc. This fraction multiplies the entire circumference of the circle to find the length of the arc.

$$\left(\frac{\text{arc angle a1}}{360}\right) \cdot (\text{Circumference } \odot \text{AB})$$

16. Construct $\overline{AB}$ and midpoint C. Construct circle CB. Construct $\overline{AD}$ and $\overline{BD}$, where point D is on the circle. Triangle ADB is a right triangle because D is inscribed in a semicircle.



Arcs and Angles

For GSP5

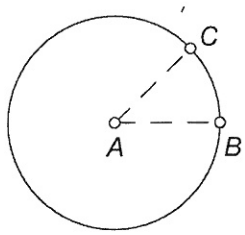
An angle with its vertex at the center of a circle is called a *central angle.* An angle whose sides are chords of a circle and whose vertex is on the circle is called an *inscribed angle.* In this activity you'll investigate relationships among central angles, inscribed angles, and the arcs they intercept.

SKETCH AND INVESTIGATE

1. Construct circle AB.

Select the segment and choose **Display | Line Style | Dashed.**

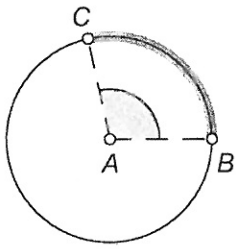
2. Construct $\overline{AB}$ and make this segment dashed.
3. Construct $\overline{AC}$, where point C is a point on the circle.



You've just created central angle BAC. Points B and C divide the circle into two arcs. The shorter arc is called a *minor arc* and the longer one is called a *major arc.* A minor arc is named after its endpoints. In the figure above right, the central angle BAC intercepts $\overset{\frown}{BC}$, where $\overset{\frown}{BC}$ is the minor arc.

Select, in order, point *B*, point *C*, and the circle. Then, in the Construct menu, choose **Arc on Circle.**

4. Construct the arc on the circle from point B to point C. While the arc is selected, make it thick.
5. Drag point C around the circle to see how it controls the arc. When you're finished experimenting, locate point C so that the thick arc is a minor arc.



Select $\overset{\frown}{BC}$; then, in the Measure menu, choose **Arc Angle.**

6. Measure the arc angle of $\overset{\frown}{BC}$.

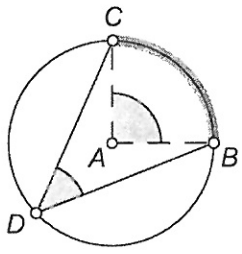
Drag the **Marker** tool from center point *A* into the angle. Then select the angle marker and choose **Measure | Angle.**

7. Measure $\angle BAC$.
8. Drag point C around the circle again and observe the measures. Pay attention to the differences when the arc is a minor arc and when it is a major arc.

Q1 Write a conjecture about the measure of the central angle and the measure of the minor arc it intercepts.

Q2 Write a conjecture about the measure of the central angle and the measure of the major arc.

9. Construct $\overline{DC}$ and $\overline{DB}$, where point D is a point on the circle, to create inscribed angle CDB.



10. Measure $\angle CDB$.

11. Drag point C and observe the measures of the arc angle and $\angle CDB$.

Q3 Write a conjecture about the measures of an inscribed angle and the arc it intercepts.

12. Drag point D (but not past point C or point B) and observe the measure of $\angle CDB$.

Q4 Write a conjecture about all the inscribed angles that intercept the same arc.

13. Drag point C so that the thick arc is as close to being a semicircle as you can make it.

14. Drag point D and observe the measure of $\angle CDB$.

Q5 Write a conjecture about angles inscribed in a semicircle.

EXPLORE MORE

15. In a new sketch, construct a circle and an arc on the circle. Measure the circumference of the circle, the arc angle, and the arc length. Use the circumference and arc angle measurements to calculate an expression equal to the arc length. Explain what you did.

16. Use your conjecture in Q5 to come up with a method for constructing a right triangle. Describe your method.